A new approach to

Odd Khovanov Homology

slides on my website!

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Berlin-Brandenburg workshop

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1

What is…
odd Khovanov homology?

L

$\left\{\text{links in } \mathbb{R}^3\right\}$

$Kh_{i,j}(L)$

(even) Khovanov homology

$\left\{\begin{array}{c}\text{bigraded abelian}\\ \text{groups}\end{array}\right\}$

-
-
-

$$L$$

$\{$links in $\mathbb{R}^3\}$

homological

quantum

$$Kh_{i,j}(L)$$

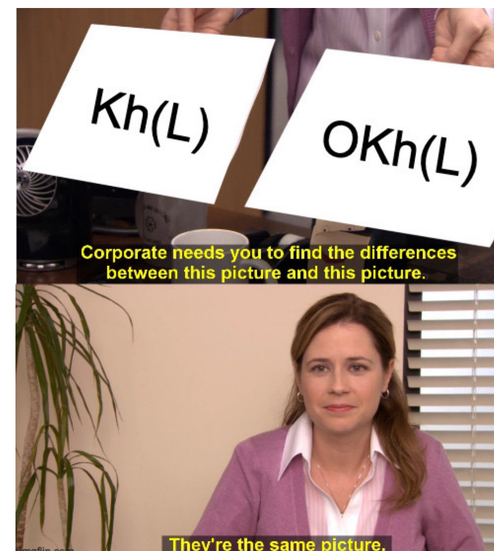
(even) Khovanov homology

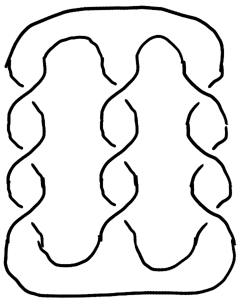
$$OKh_{i,j}(L)$$

odd Khovanov homology

$\{$bigraded abelian groups$\}$

- $\mathrm{Jones}(L) = \chi_q(Kh(L)) = \chi_q(OKh(L))$
- same skein exact sequence!
- $Kh(L, \mathbb{F}_2) \cong OKh(L, \mathbb{F}_2)$





P(3,3,-3)

$j \downarrow \; \rightarrow i$

even

	-6	**-5**	**-4**	**-3**	**-2**	**-1**	**0**
0							**2**
-2						**1**	
-4					**1**		
-6				**2**			
-8			**1**				
-10		**1**					
-12	**1**						

odd

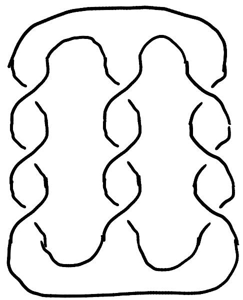
	-6	**-5**	**-4**	**-3**	**-2**	**-1**	**0**
0							**2**
-2						**1**	$\boxed{1_3}$
-4					**1**		
-6				**2**			
-8			**1**				
-10		**1**					
-12	**1**						

[Shumakovitch 11]

Theorem (Ebert–S. 25).

Theorem (Migdail–Wehrli 24).

Conjecture (Oszváth–Rasmussen–Szabó 07).



P(3,3,-3)

even

	-6	-5	-4	-3	-2	-1	0
0							2
-2						1	
-4					1		
-6				2			
-8			1				
-10		1					
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odd

	-6	-5	-4	-3	-2	-1	0
0							2
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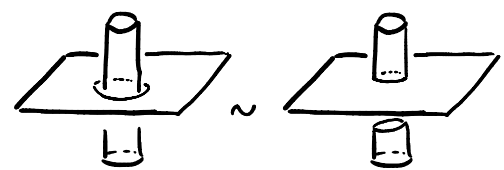
-4					1		
-6				2			
-8			1				
-10		1					
-12	1						

[Shumakovitch 11]

3-torsion!

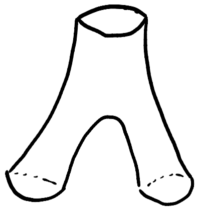
Theorem (Ebert–S. 25). $OKh(P(n,n,-n))$ has n-torsion.

Theorem (Migdail–Wehrli 24). Odd Khovanov homology detects ribbon moves.

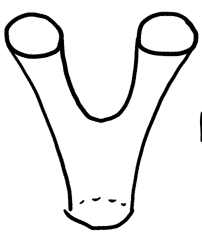


Conjecture (Oszváth–Rasmussen–Szabó 07). $OKh(L) \Rightarrow \widehat{HF}(\Sigma(L))$

even | odd

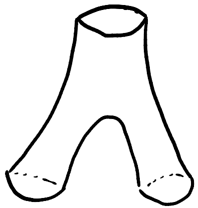
 $\longmapsto$

$$\mathbb{Z}[x_1, x_2]/x_1^2, x_2^2 \xrightarrow{P|_{x_1, x_2 \mapsto x}} \mathbb{Z}[x]/x^2$$

 $\longmapsto$

$$\mathbb{Z}[x]/x^2 \xrightarrow{(x_1 + x_2) P|_{x \mapsto x_1}} \mathbb{Z}[x_1, x_2]/x_1^2, x_2^2$$

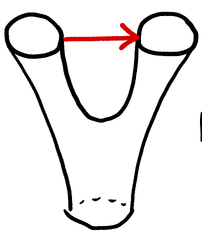
even | odd



$\mapsto$

$$\mathbb{Z}[x_1, x_2]/x_1^2, x_2^2 \xrightarrow{p|_{x_1, x_2 \mapsto x}} \mathbb{Z}[x]/x^2$$

$$\Lambda(x_1, x_2) \xrightarrow{p|_{x_1, x_2 \mapsto x}} \Lambda(x)$$



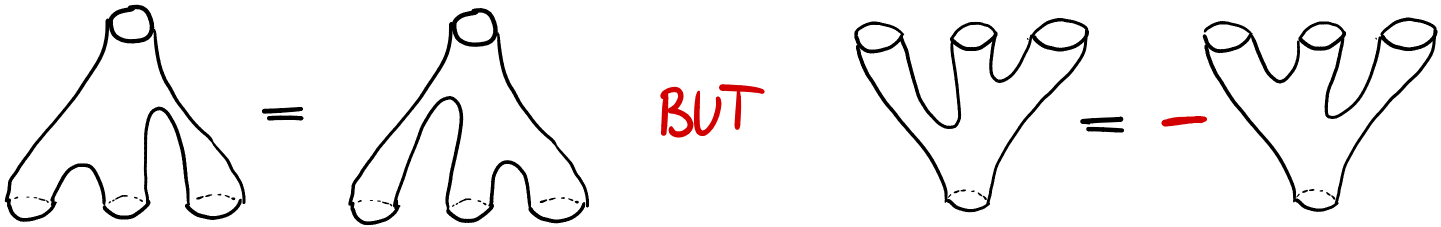
$\mapsto$

$$\mathbb{Z}[x]/x^2 \xrightarrow{(x_1 + x_2)\, p|_{x \mapsto x_1}} \mathbb{Z}[x_1, x_2]/x_1^2, x_2^2$$

$$\Lambda(x) \xrightarrow{(x_1 - x_2) \wedge p|_{x \mapsto x_1}} \Lambda(x_1, x_2)$$

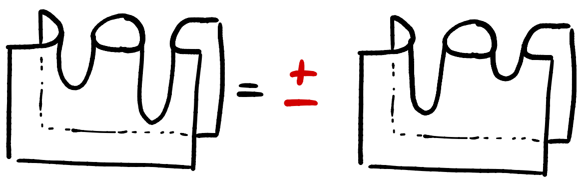
$$\Lambda(x_1, \ldots, x_n) = \mathbb{Z}\langle x_1, \ldots, x_n \rangle / (x_i x_j = -x_j x_i,\ x_i^2 = 0)$$

Theorem (Oszváth–Rasmussen–Szabó 07).



Theorem (Oszváth–Rasmussen–Szabó 07). Still gives a link invariant!

Q1. Extension to tangles?



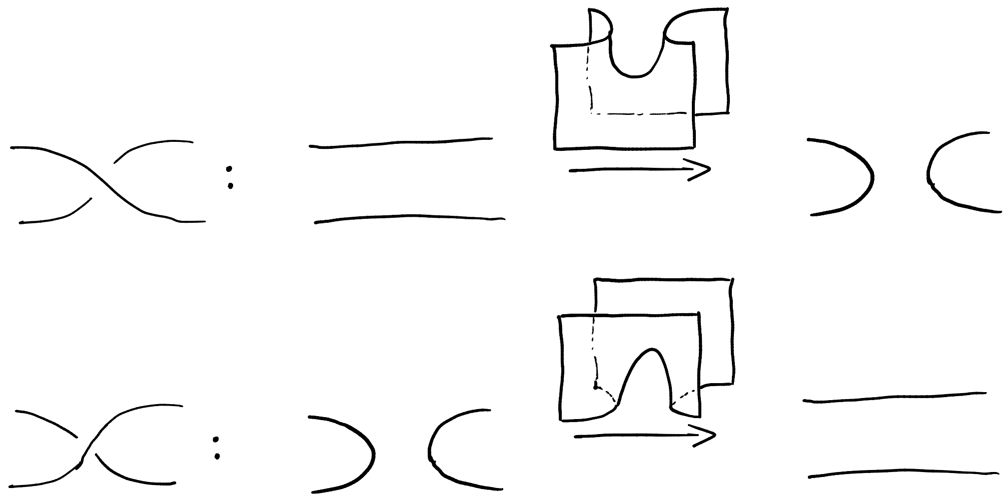
not local!

Q2. What is an "**odd**" link homology, *really*?

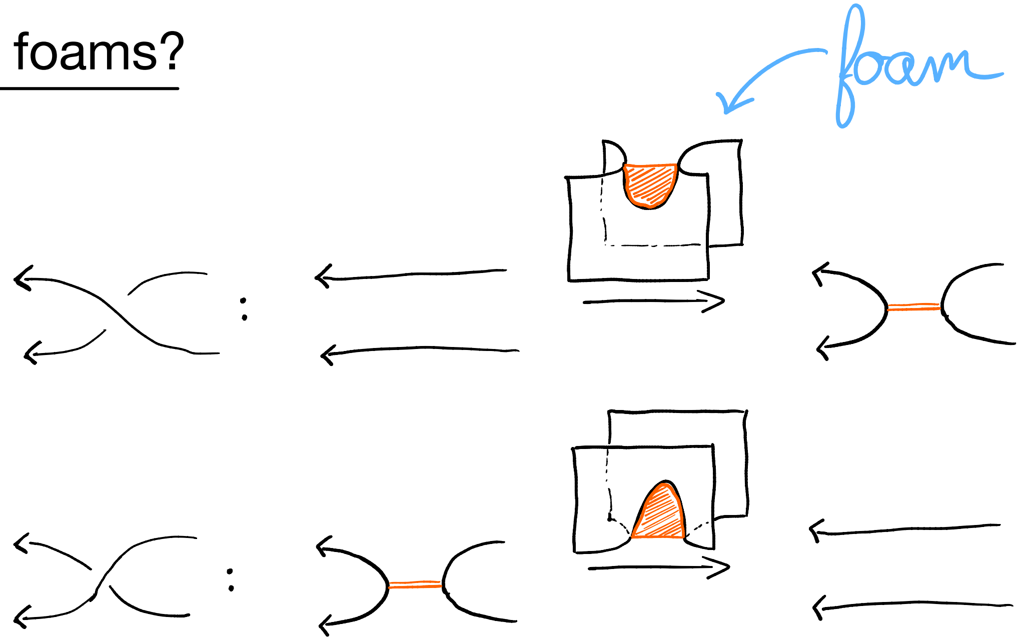
2

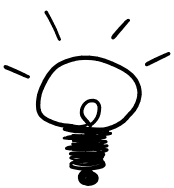
A novel approach: super foams

What are… foams?

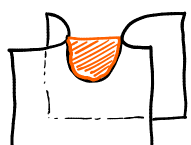


What are… foams?



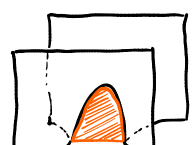


two saddles



even

and

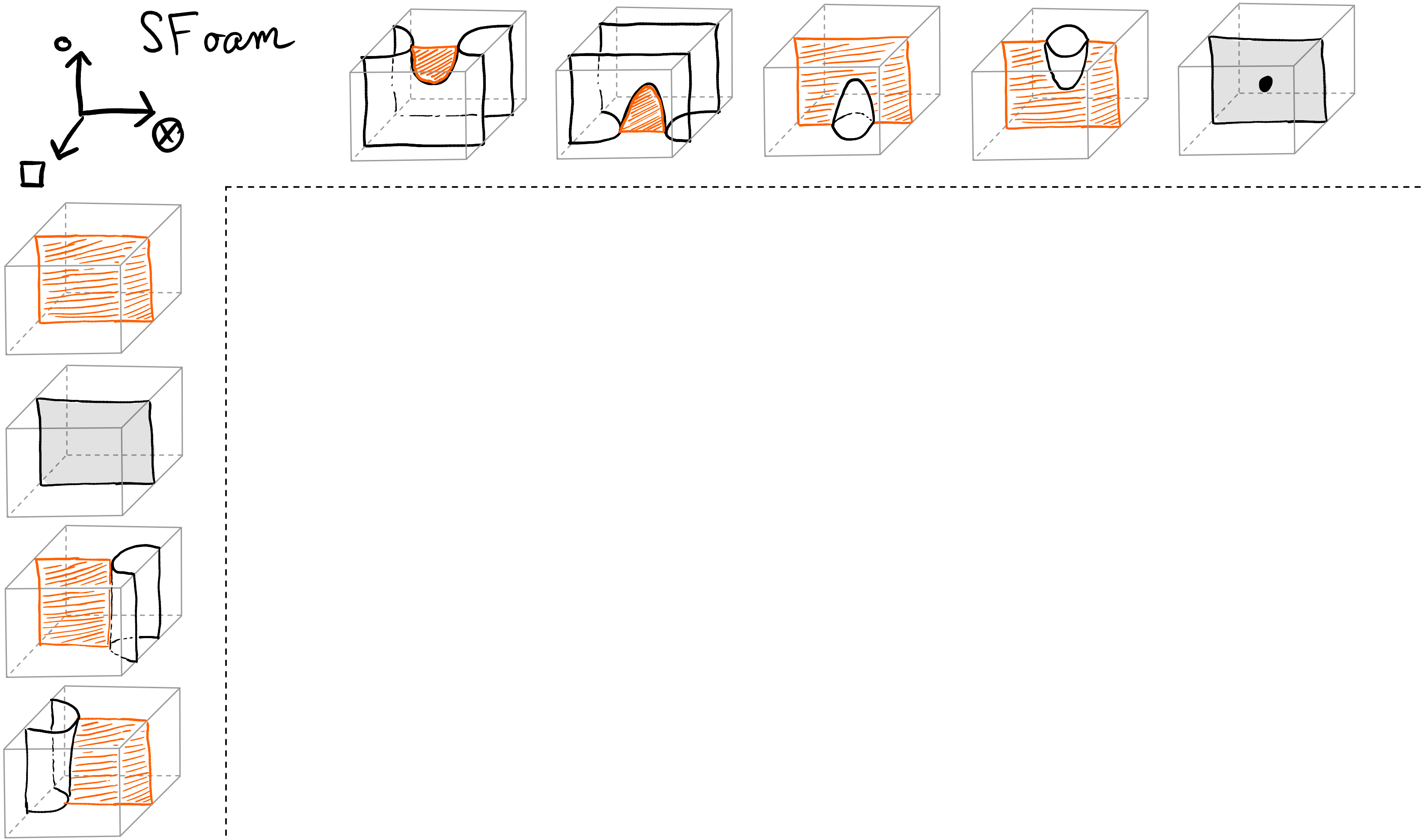


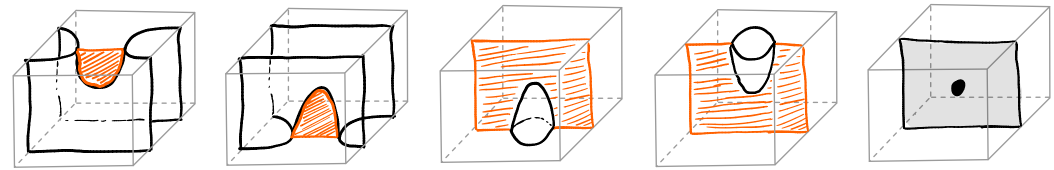
odd

$\in$

the monoidal 2-supercategory

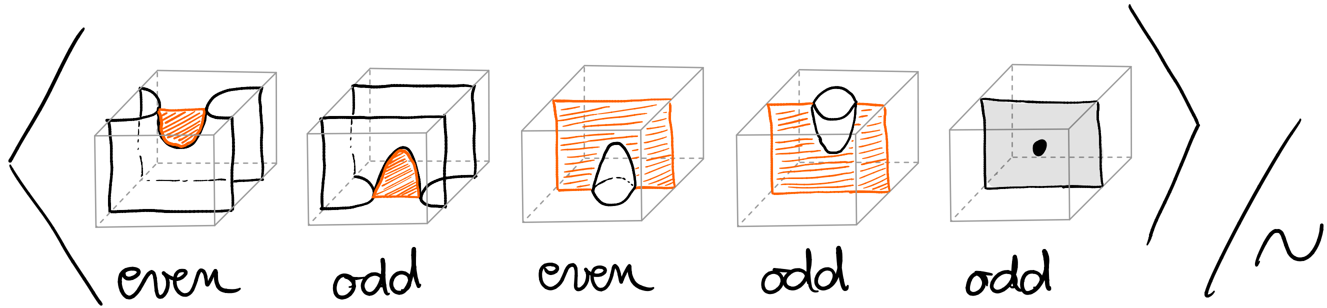
SFoam


SFoam

$\mathrm{SFoam} = \Big\langle$  $\Big\rangle \Big/ \sim$

Theorem (S. 25).

SFoam =



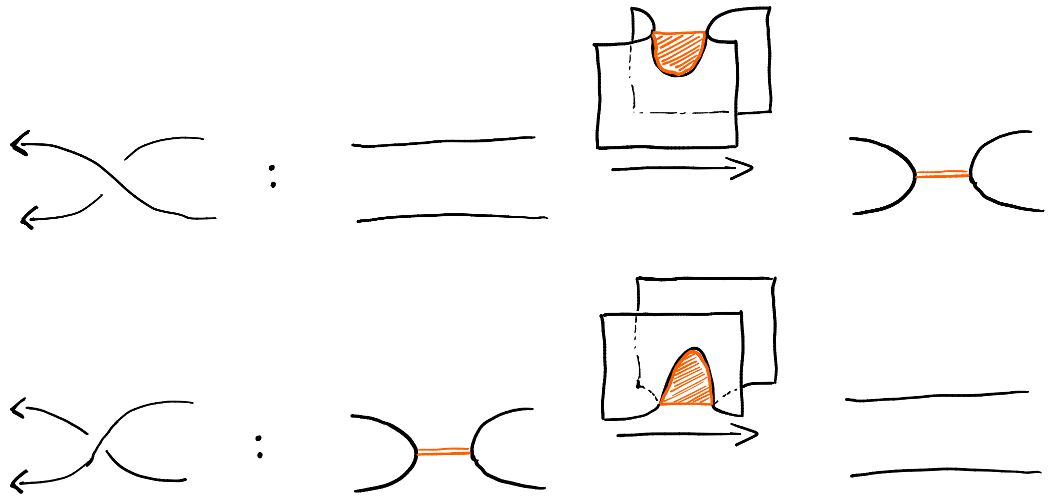
As a **monoidal 2-supercategory**! ("higher superalgebra")

$$\begin{array}{|c|c|}\hline \alpha & \mathrm{ID} \\ \hline \mathrm{ID} & \beta \\ \hline \end{array} = (-1)^{|\alpha||\beta|} \begin{array}{|c|c|}\hline \mathrm{ID} & \alpha \\ \hline \beta & \mathrm{ID} \\ \hline \end{array}$$

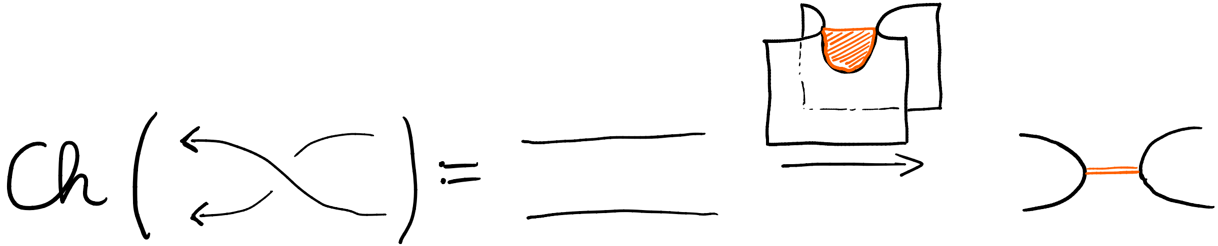
super interchange law

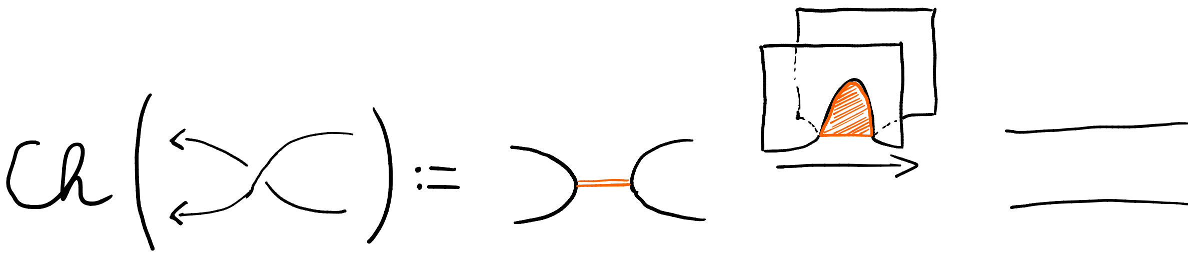
Theorem (S. 25). Homspaces in **SFoam** have the expected dimension.

Definition.



Definition. For β a braid, let $\mathrm{Ch}(\beta) \in \mathrm{Ch}(\mathrm{SFoam})$:



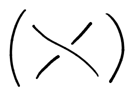
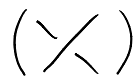


and $\mathrm{Ch}(\beta_1 \otimes \dots \otimes \beta_n) = \mathrm{Ch}(\beta_1) \otimes \dots \otimes \mathrm{Ch}(\beta_n)$.

super $\otimes$ of chain complexes (S. 20)

Theorem (S.-Vaz 23).

Theorem (S.-Vaz 23).
This recovers odd Khovanov homology

SLOGAN. Odd Khovanov homology arises from the super interaction of even  and odd  chain complexes.